Online Appendix

A Omitted Proofs

Proof of Lemma 1. First I show that global incentive compatibility implies the conditions in the lemma. Global incentive compatibility implies that

$$\begin{aligned}\Pi(\hat{s}|\hat{s}) &\geq \Pi(s|\hat{s}) \\ &= \Pi(s|s) + \Pi(s|\hat{s}) - \Pi(s|s) \\ &= \Pi(s|s) - m(\tilde{\theta}(s))\tilde{q}(s)(\hat{v} - v)\end{aligned}$$

Switching $\hat{s}$ and s and combining inequalities gives

$$-m(\tilde{\theta}(s))\tilde{q}(s)(\hat{v} - v) \leq \Pi(\hat{s}|\hat{s}) - \Pi(s|s) \leq -m(\tilde{\theta}(\hat{s}))\tilde{q}(\hat{s})(\hat{v} - v) \tag{A.1}$$

Part (i): Clearly, if $\hat{v} = v$, then $\Pi(\hat{s}|\hat{s}) = \Pi(s|s)$, regardless of $\hat{f}$ and f. Therefore, $\Pi((f, v)|(f, v))$ is constant in f, and is fully determined by v.

Part (ii): For shorthand, write $\Pi((f, v)|(f, v)) = \Pi(v)$, and rewrite (A.1) as

$$-m(\tilde{\theta}(s))\tilde{q}(s)(\hat{v} - v) \leq \Pi(\hat{v}) - \Pi(v) \leq -m(\tilde{\theta}(\hat{s}))\tilde{q}(\hat{s})(\hat{v} - v). \tag{A.2}$$

The right hand inequality indicates that if $\hat{v} > v$, then $\Pi(\hat{v}) \leq \Pi(v)$, so Π is decreasing in v. Next, for any $\epsilon > 0$, choose an arbitrary set of disjoint intervals (a_k, b_k) in $V = [\underline{v}, \bar{v}]$ such that $\sum_{k=1}^{N}(b_k - a_k) < \epsilon$ and set of arbitrary asset qualities $\{f_k\}$ so that $f_k \in \tilde{S}(a_k)$. Then

$$\begin{aligned}\sum_{k=1}^{N}|\Pi(b_k) - \Pi(a_k)| &= -\sum_{k=1}^{N}(\Pi(b_k) - \Pi(a_k)) \\ &\leq \sum_{k=1}^{N} m(\tilde{\theta}(f_k, a_k))\tilde{q}(f_k, a_k)(b_k - a_k) \\ &\leq \sum_{k=1}^{N}(b_k - a_k) < \epsilon,\end{aligned}$$

so Π is absolutely continuous, and is therefore differentiable almost everywhere. Now if $\Pi'(v)$ exists, then dividing (A.2) by $(\hat{v} - v)$ and letting $\hat{v}$ go to v from above and below gives (ENV).

Part (iii): This follows directly from (A.2).

I now show that the conditions in the lemma imply global incentive compatibility. Denote $m(\tilde{\theta}(f, v))\tilde{q}(f, v)$ by $H(f, v)$. Next, take $\{\hat{s}, s\} \equiv \{(\hat{f}, \hat{v}), (f, v)\} \subset \tilde{S}$, and let $h(\cdot)$ be any function over $[v, \hat{v}]$ with $h(v) = f$, $h(\hat{v}) = \hat{f}$, and such that the graph of h is contained in $\tilde{S}$. The lemma's third condition guarantees that $H(h(\cdot), \cdot)$ is decreasing, and the second condition indicates that where Π' exists, which is almost everywhere, then $-H(h(v), v) = \Pi'(v)$. I now have

$$\begin{aligned}\Pi(\hat{s}|\hat{s}) = \Pi(\hat{v}) &= \Pi(v) + \int_v^{\hat{v}} \Pi'(t)dt = \Pi(v) - \int_v^{\hat{v}} H(h(t), t)dt \\ &\geq \Pi(v) - \int_v^{\hat{v}} H(f, v)dt = \Pi(v) - m(\tilde{\theta}(s))\tilde{q}(s)(\hat{v} - v) \\ &= \Pi(s|\hat{s}),\end{aligned}$$

where the second equality follows from the absolute continuity of Π, and the inequality follows from the fact that $H(h(\cdot), \cdot)$ is decreasing. Therefore, global incentive compatibility holds. □

Corollary A.1. *If $m(\tilde{\theta}(f, v))\tilde{q}(f, v)$ is continuous in v at some value, then $\Pi(\cdot)$ is differentiable at the same value, and therefore $\Pi'(\cdot) = m(\tilde{\theta}(f, \cdot))\tilde{q}(f, \cdot))$ at that value.*

Proof. Divide (A.2) by $\hat{v} - v$, and let $\hat{v}$ converge to v above and below. Then apply Lemma 1. □

Lemma A.1. *In equilibrium, all sellers receive strictly positive expected utility, and therefore all sellers attempt to trade with buyers.*

Proof. Suppose to the contrary that there exists some seller $(f, v) \in \tilde{S}$ that receives zero expected utility, and let v_i be the infimum of seller values v that receive zero expected utility in equilibrium. Then $\max_{(p,q) \in M} m(\theta(p, q))q(p -$

$v_i) \leq 0$. Recall that for any $(p,q) \in M$, $\theta(p,q) > 0$ and buyer free-entry is satisfied, so $q > 0$. Therefore, the supremum p_s of prices posted in equilibrium must satisfy $p_s \leq v_i$. But then buyers can post off-equilibrium $p' \in (v_i, \underline{f}(v_i) - k)$ and $q' = 1$, and expect some type in the set $\{(f,v) \in \tilde{S} : v \geq v_i\}$. But then $E[\tilde{f}|p',1] \geq \underline{f}(v_i)$ and $\theta(p',1) = 0$. So then $n(\theta(p',q'))q'(E[\tilde{f}|p',q'] - p') = n(0) \cdot 1 \cdot (E[\tilde{f}|p',1] - p') = E[\tilde{f}|p',1] - p' \geq \underline{f}(v_i) - p' > k$. So $(p',1)$ is a profitable deviation, which is a contradiction. □

Proof of Lemma 3. By Lemma 1, $\Pi(f,\delta)$ is a function only of the product $v = \delta f$, and therefore, $\theta(p,q,f,\delta)$ also depends on f and δ only through the product $v = \delta f$. So we can write $\theta(p,q,v)$. By the definition of $\theta(p,q,v)$, it is the unique value θ which solves $m(\theta)q(p-v) = \Pi(v)$ if $q(p-v) > \Pi(v)$ and ∞ otherwise. By Lemma 1, $\Pi(\cdot)$ is continuous and convex, and therefore has left and right derivatives Π'_- and Π'_+. So $\theta(p,q,\cdot)$ is also continuous and has left and right derivatives θ_{v-} and θ_{v+} over $\{v \in V : \Pi(v)/[q(p-v)] < 1\}$. Taking the right derivative of $m(\theta(p,q,v))q(p-v) = \Pi(v)$ with respect to v gives

$$\theta_{v+}m'(\theta(p,q,v))q(p-v) - m(\theta(p,q,v))q = \Pi'_+(v) \geq -m(\theta(f,v))q(f,v),$$

where the inequality follows from (A.2). Rearranging the above expression gives $\theta_{v+}m'(\theta(p,q,v))q(p-v) \geq m(\theta(p,q,v))q - m(\theta(f,v))q(f,v) = \Pi(v)/(p-v) - \Pi(v)/(p(f,v)-v) = [p(f,v)-p]\Pi(v)/[(p-v)(p(f,v)-v)]$. So if $p < p(f,v)$, then $\theta_{v+}(p,q,v) > 0$. A similar argument shows that if $p > p(f,v)$, then $\theta_{v-}(p,q,v) < 0$.

(i) If $p > p(f,v)$ for all $(f,v) \in \tilde{S}$, then $\theta_{v-}(p,q,v) < 0$ for all $v \in V$, so $\arg\min_v \theta(p,q,v) = \bar{v}$.

(ii) If $p < p(f,v)$ for all $(f,v) \in \tilde{S}$, then $\theta_{v-}(p,q,v) > 0$ for all $v \in V$, so $\arg\min_v \theta(p,q,v) = \underline{v}$.

(iii) Let $A = \{v \in V : p(f,v) = p \text{ for some } f \in \tilde{S}(v)\}$. Because $p(f,v)$ is weakly increasing in v, A is connected. If v' exceeds all $v \in A$, then for all $f' \in \tilde{S}(v')$, it must be that $p(f',v') > p$, so $\theta_{v+}(p,q,v') > 0$. If v' is less than all $v \in A$, then for all $f' \in \tilde{S}(v')$, it must be that $p(f',v') < p$, so $\theta_{v-}(p,q,v') < 0$. Therefore, $\arg\min_{v'} \theta(p,q,v') \subset A$.

(iv) If p lies between equilibrium prices, then there must be some v such that the price function $p(f,v)$ jumps over p at v. Because p is weakly increasing in v, for all (f',v') such that $v'>v$ and $f'\in\tilde{S}(v')$, it must be that $p(f',v')>p$, so $\theta_{v+}(p,q,v')>0$; and for all (f',v') such that $v'<v$ and $f'\in\tilde{S}(v')$, it must be that $p(f',v')<p$, so $\theta_{v-}(p,q,v')<0$. As a result, $\arg\min_{v'}\theta(p,q,v')=v$. □

Proof of Lemma 5. Because $p(\cdot)$ is increasing (Lemma 5), $p(\cdot)$ is left- and right-continuous everywhere. Let p_+ denote the right limit of $p(\cdot)$ at $\hat{f}$, and of course p is the left limit.

Suppose to the contrary that $p(\cdot)$ is discontinuous at $\hat{f}$. Then $p<p_+$ and $p(\hat{f})\in[p,p_+]$. Suppose a buyer considers deviating by posting $p'\in(p,p_+)\setminus\{p(\hat{f})\}$ and $q'=1$. Then by Lemma 3, buyers expect type $\hat{f}$, so $\theta(p',1)=\theta(p',1,\hat{f})$. By the continuity of $\Pi(\cdot)$, $\Pi(\hat{f})=\lim_{f\uparrow\hat{f}}\Pi(f)=C(p-\delta\hat{f})$. So then $m(\theta(p',1,\hat{f}))\cdot 1\cdot(p'-\delta\hat{f})=\Pi(\hat{f})=C(p-\delta\hat{f})$, and therefore $m(\theta(p',1))=m(\theta(p',1,\hat{f}))<C=m(\theta(f))q(f)\leq m(\theta(f))$ for all $f\in A$, which implies $\theta(p',1)<\theta(f)$ for all $f\in A$.

Observe that there exists an $f\in A$ such that $h(f)<\hat{f}$, so $n(\theta(p',q'))q'(E[\tilde{f}|p',q']-p)=n(\theta(p',1))[\hat{f}-p]>n(\theta(f))q(f)(h(f)-p)=k$. So for small enough $p'>p$, $n(\theta(p',q'))q'(E[\tilde{f}|p',q']-p')>n(\theta(f))q(f)(h(f)-p)=k$, and therefore $(p',1)$ is a profitable deviation. □

Proof of Lemma 6. Let $I(p')=\{f\in[\underline{f},\bar{f}]:p(f)=p'\}$ and let $\rho(\epsilon)=\inf_{f\in B(\epsilon)}[\sup(I(p(f)))-\inf(I(p(f)))]$; that is, $\rho(\epsilon)$ is the infimum of the lengths of intervals in $B(\epsilon)$ on which $p(\cdot)$ is constant. I claim that for all $\epsilon>0$, $\rho(\epsilon)=0$. If not, then for all $f\in(\hat{f},\hat{f}+\rho(\epsilon))$, $p(f)$ is constant and strictly greater than p, which contradicts the continuity of $p(\cdot)$ at $\hat{f}$.

That $\rho(\epsilon)=0$ for all $\epsilon>0$ implies $\sup_{f\in B(\epsilon)}q(f)=1$. To see this, suppose to the contrary that there exists $q'<1$ such that $q(f)<q'$ for all $f\in B(\epsilon)$. But consider the deviation $(p',1)$, where p' is chosen so that the length of $I(p')$ is arbitrarily small. Then by Lemma 3, $E[\tilde{f}|p',1]$ is arbitrarily close to $h(f)$ for any $f\in I(p')$. Also, by Lemma 3, there exists an $f\in I(p')$, such that $\theta(p',1)=\theta(p',1,f)$. Observe that

$m(\theta(p', 1, f))(p' - \delta f) = \Pi(f) = m(\theta(f))q(f)(p' - \delta f)$, which implies that $m(\theta(p', 1, f)) = m(\theta(f))q(f) < m(\theta(f))$, and therefore $\theta(p', 1) = \theta(p', 1, f) < \theta(f)$. So then $n(\theta(p', 1))(E[\tilde{f}|p', 1] - p') > n(\theta(f))q(f)(h(f) - p') = k$, so $(p', 1)$ is a profitable deviation. □

Detailed proof of Theorem 1. The in-print appendix points out that after establishing that the price $p(\cdot)$ is strictly increasing in asset quality, the proof of Theorem 2 may be applied to the case of one-dimensional private information to show that for all $f \in [\underline{f}, \bar{f}]$, $q(f) = 1$, that $\theta(f)$ is continuous and satisfies the differential equation in the proposition, and buyers cannot profitably deviate. Here I include detailed proofs of these remaining steps.

Lemma A.2. *For all $f \in [\underline{f}, \bar{f}]$, $q(f) = 1$.*

Proof. Suppose to the contrary that there exists an $f \in [\underline{f}, \bar{f}]$ such that $q(f) < 1$. By Proposition 5, $p(\cdot)$ is invertible, so type f is the only type which chooses price $p(f)$. As a result, if buyers post off-equilibrium terms $(p(f), 1)$, by Lemma 3 they expect type f. Their expected market tightness $\theta(p(f), 1)$ is given by $\theta(p(f), 1, f)$, which solves $m(\theta(p(f), 1))(p(f) - \delta f) = \Pi(f) = m(\theta(f))q(f)(p(f) - \delta f)$, so $m(\theta(p(f), 1)) = m(\theta(f))q(f) < m(\theta(f))$, and therefore $\theta(p(f), 1) < \theta(f)$. Buyer's expected profit from posting $(p(f), 1)$ is $n(\theta(p(f), 1))(f - p(f)) > n(\theta(f))q(f)(f - p(f)) = k$, so $(p(f), 1)$ is a profitable deviation, contradicting equilibrium. □

Given Propositions 5 and A.2, and buyer zero-profit, we must have $\Pi(f) = m(\theta(f))(1 - \delta)f - k\theta$. Let $\Pi_{CI}(f) = m(\theta_{CI}(f))(1 - \delta)f - k\theta_{CI}(f)$, where $\theta_{CI}(f)$ is the unique maximizer of $m(\theta)(1 - \delta)f - k\theta$. Then given $f \in [\underline{f}, \bar{f}]$, for any $\Pi \in (0, \Pi_{CI}(f))$, the concavity of $m(\theta)(1 - \delta)f - k\theta$ implies that there exist exactly two values of θ that solve $\Pi = m(\theta)(1 - \delta)f - k\theta$, which I denote by $\theta_U(\Pi, f)$ and $\theta_D(\Pi, f)$, where $\theta_U(\Pi, f) > \theta_D(\Pi, f)$.

Next, I show that $\theta(f)$ is continuous for all $f > \underline{f}$. If $\theta(f)$ has a discontinuity, then by Lemma 2, it must jump downward. By the continuity of $\Pi(\cdot)$, it must jump from $\theta_U(\Pi(f), f)$ to $\theta_D(\Pi(f), f)$, which can only occur once. If the jump occurs at some $f' > \underline{f}$, then for all $f < f'$, $\theta(f)$

is continuous and equal to $\theta_U(\Pi(f), f)$, and by Corollary A.1, $\Pi(\cdot)$ is differentiable in this range, which implies so is $\theta(f) = \theta_U(\Pi(f), f)$. We then have $\Pi'(f) = [m'(\theta(f))(1-\delta)f - k\theta(f)]\theta'(f) + m(\theta(f))(1-\delta)$, which implies $[m'(\theta(f))(1-\delta)f - k\theta(f)]\theta'(f) = \Pi'(f) - m(\theta(f))(1-\delta) < 0$. Because $\theta(f) = \theta_U(\Pi(f), f)$, it must be that $m'(\theta(f))(1-\delta)f - k < 0$, so $\theta'(f) > 0$, contradicting Lemma 2. So for all $f > \underline{f}$, $\theta(f)$ is continuous and equal to $\theta_D(\Pi(f), f)$. By Corollary A.1, $-\delta m(\theta(f)) = \Pi'(f) = [m'(\theta(f))(1-\delta)f - k\theta(f)]\theta'(f) + m(\theta(f))(1-\delta)$, which gives the differential equation in the proposition.

To show the boundary condition $\theta(\underline{f}) = \theta_{CI}(\underline{f})$, suppose not. I first show that $p_{CI}(\underline{f}) = \underline{f} - k/n(\theta_{CI}(\underline{f})) \neq p(f)$ for all f, establishing that $p_{CI}(\underline{f})$ is a deviation. First observe that for all $f > \underline{f}$, $\theta(f) = \theta_D(\Pi(f), f) \leq \theta_{CI}(f)$. So by the monotonicity of $\theta(f)$ and the continuity of $\theta_{CI}(f)$, it must be that $\theta(f) \leq \lim_{f \to \underline{f}} \theta(f) \leq \theta_{CI}(\underline{f})$ for all $f > \underline{f}$. But then for all $f > \underline{f}$, $p_{CI}(\underline{f}) = \underline{f} - k/n(\theta_{CI}(\underline{f})) < f - k/n(\theta(f)) = p(f)$, and of course $p_{CI}(\underline{f}) \neq p(\underline{f})$, so $p_{CI}(\underline{f})$ is a deviation.

If buyers post $p' = p_{CI}(\underline{f})$ and $q' = 1$, then by Lemma 3, they expect type $\underline{f}$, so $\theta(p', 1)$ satisfies $m(\theta(p', 1))(p' - \delta\underline{f}) = \Pi(\underline{f}) < \Pi_{CI}(\underline{f}) = m(\theta_{CI}(\underline{f}))(p_{CI}(\underline{f}) - \delta\underline{f})$. This implies $\theta(p', 1) < \theta_{CI}(\underline{f})$. But then buyers expect profit $n(\theta(p', 1))[\underline{f} - p_{CI}(\underline{f})] > n(\theta_{CI}(\underline{f}))[\underline{f} - p_{CI}(\underline{f})] = k$, so $(p_{CI}(\underline{f}), 1)$ is a profitable deviation, contradicting equilibrium. Therefore, $\theta(\underline{f}) = \theta_{CI}(\underline{f})$.

I now check that buyers do not have a profitable off-equilibrium deviation. First observe the equilibrium prices p occupy the same space $[\underline{p}, \bar{p}]$ as in the one-signal case, but equilibrium q is equal to 1 everywhere, so $M^* = [\underline{p}, \bar{p}] \times 1$. First observe that $m(\theta(p, q, f))q(p - \delta f) = \Pi(f)$. Following the same reasoning as in the case with one signal, $\theta_3(p, q, f)$ has the same sign as $(p^*(f) - \delta f)/(p - \delta f) - 1$. So if buyers deviate with $(p, q) \in [\underline{p}, \bar{p}] \times (0, 1)$, then $\theta(p, q, f)$ is minimized by the unique $f \in [\underline{f}, \bar{f}]$ that chooses p in equilibrium; i.e., $p^*(f) = p$. Therefore, $\theta(p, q) = \theta(p, q, p^{*-1}(p))$, and I have $m(\theta(p, q))q(p - \delta f) = \Pi(f) = m(\theta^*(f))(p^*(f) - f)$, where f is the type which chooses $p^*(f) = p$ in equilibrium (and therefore $(p - \delta f)$ and $(p^*(f) - \delta f)$ cancel). This gives $m(\theta(p, q)) =$

$m(\theta^*(f))/q > m(\theta^*(f))$, because $q < 1$. Therefore, $\theta(p,q) > \theta^*(f)$, and because $n(\theta) = m(\theta)/\theta$ is strictly decreasing, I have $k = n(\theta^*(f))(f - p^*(f)) = n(\theta^*(f))(f-p) > n(\theta(p,q))q(f-p)$, so $(p,q) \in [\underline{p}, \bar{p}] \times (0,1)$ is not a profitable deviation. Next, suppose a coalition of buyers post a price $p > \bar{p}$ and $q \in [0,1]$. First recall that ;because $\theta_3(p,q,f)$ has the same sign as $(p^*(f) - \delta f)/(p - \delta f) - 1$, for $p > \bar{p} \equiv p^*(\bar{f})$, $\theta(p,q,f)$ must be minimized by $\bar{f}$. So buyers expect type $\bar{f}$, and $\theta(p,q) = \theta(p,q,\bar{f}$ for $p > \bar{p}$. Next, observe that $\theta(p,1)$ satisfies $m(\theta(p,1))(p - \delta\bar{f}) = \Pi(f) = m(\theta^*(\bar{f}))(p^*(\bar{f}) - \delta\bar{f})$, so then $\theta(p,1) < \theta^*(\bar{f})$. Because $\theta^*(\bar{f})$ is less than the complete information θ and $\theta(p,1)$ is even lower, I must have $m(\theta^*(\bar{f})(\bar{f} - \delta\bar{f}) - k(\theta^*(\bar{f})) > m(\theta(p,1))(\bar{f} - \delta\bar{f}) - k\theta(p,1)$. So the zero-profit condition of buyers who trade with $\bar{f}$ in equilibrium gives

$$
\begin{aligned}
0 &= m(\theta^*(\bar{f}))(\bar{f} - p^*(\bar{f})) - k\theta^*(\bar{f}) \\
&= m(\theta^*(\bar{f}))(\bar{f} - \Pi(\bar{f})/m(\theta^*(\bar{f})) - \delta\bar{f}) \\
&= -\Pi(\bar{f}) + m(\theta^*(\bar{f}))(\bar{f} - \delta\bar{f}) - k\theta^*(\bar{f}) \\
&> -\Pi(\bar{f}) + m(\theta(p,1))(\bar{f} - \delta\bar{f}) - k\theta(p,1) \\
&= -m(\theta(p,1))(p - \delta\bar{f}) + m(\theta(p,1))(\bar{f} - \delta\bar{f}) - k\theta(p,1) \\
&= m(\theta(p,1))(\bar{f} - p) - k\theta(p,1)
\end{aligned}
$$

Dividing both sides by $\theta(p,1)$ gives $0 > n(\theta(p,1))(\bar{f} - p) - k$. Next, note that for $q \in [0,1]$, $m(\theta(p,q))q(p - \delta\bar{f}) = m(\theta(p,1))(p - \delta\bar{f})$, so $\theta(p,q) \geq \theta(p,1)$, and I have $n(\theta(p,q))q(\bar{f} - p) - k$, so $(p,q) \in (\bar{p}, \infty) \times [0,1]$ is not a profitable deviation. Finally, suppose that a coalition posts $p < \underline{p}$ with $q \in [0,1]$. Again, because $\theta_3(p,q,f)$ has the same sign as $(p^*(f) - \delta f)/(p - \delta f) - 1$, if (p,q) attracts any type, it must be $\underline{f}$, so $\theta(p,q) \equiv \theta(p,q,\underline{f})$. Recall that $\underline{f}$ receives the complete information liquidity, and because $m(\theta(p,q))q(p - \delta\underline{f}) = \Pi(f) = m(\theta^*(\underline{f}))(p^*(f) - \delta\underline{f})$ and therefore $\theta(p,q) > \theta^*(\underline{f}) = \theta_{CI}(\underline{f})$, it must be that $m(\theta^*(ulf))(\underline{f} - \delta\underline{f}) - k\theta^*(\underline{f}) > m(\theta(p,q))(\underline{f} - \delta\underline{f}) - k\theta(p,q)$. So applying the same reasoning as for $p > \bar{p}$, I have $0 > n(\theta(p,q))(\underline{f} - p) - k \geq n(\theta(p,q))q(\underline{f} - p - k)$, so $(p,q) \in [0, \underline{p}) \times [0,1]$ is not a profitable deviation. $\square$

Proof of Proposition 1. Modify the equilibrium definition so that $q = 0$

does not count as equilibrium (it's the same as that buyer not participating). So in equilibrium, $q > 0$. Lemmas A.1, 2, 3, 4, 5, 6, Proposition 5, Proposition A.2, and the remaining arguments hold, so I have proven Proposition 1. □

Lemma A.3. *Given (p, q), if $p \notin [\underline{p}, \bar{p}]$, then (p, q) is not a profitable deviation.*

Proof. Suppose that a coalition posts $p > \bar{p}$, with any $q \in [0, 1]$. Then by Lemma 3, buyers expect $\bar{v} = \bar{\delta}\bar{f}$. Next, observe that $\theta(p, 1)$ satisfies $m(\theta(p, 1))(p-\bar{v}) = \Pi(\bar{v}) = m(\theta(\bar{f}, \bar{v}))(p^*(\bar{v})-\bar{v})$, where I have used $q(\bar{f}, \bar{v}) = 1$; so then, $\theta(p, 1) < \theta(\bar{f}, \bar{v})$. Because $\theta(\bar{f}, \bar{v})$ is less than the complete information θ, and $\theta(p, 1)$ is even lower, I must have $m(\theta(\bar{f}, \bar{v}))(\bar{f} - \bar{v}) - k\theta(\bar{f}, \bar{v}) > m(\theta(p, 1))(\bar{f} - \bar{v}) - k\theta(p, 1)$ So the zero-profit condition of buyers who trade with $(\bar{f}, \bar{v})$ in equilibrium gives

$$\begin{aligned}
0 &= m(\theta(\bar{f}, \bar{v}))(\bar{f} - p^*(\bar{v})) - k\theta(\bar{f}, \bar{v}) \\
&= m(\theta(\bar{f}, \bar{v}))(\bar{f} - \Pi(\bar{v})/m(\theta(\bar{f}, \bar{v})) - \bar{v}) - k\theta(\bar{f}, \bar{v}) \\
&= -\Pi(\bar{v}) + m(\theta(\bar{f}, \bar{v}))(\bar{f} - \bar{v}) - k\theta(\bar{f}, \bar{v}) \\
&> -\Pi(\bar{v}) + m(\theta(p, 1))(\bar{f} - \bar{v}) - k\theta(p, 1) \\
&= -m(\theta(p, 1))(p - v) + m(\theta(p, 1))(\bar{f} - \bar{v}) - k\theta(p, 1) \\
&= m(\theta(p, 1))(\bar{f} - p) - k\theta(p, 1).
\end{aligned}$$

Dividing both sides by $\theta(p, 1)$ gives $0 > n(\theta(p, 1))(\bar{f} - p) - k$. Next, note that for $q \in [0, 1]$, $m(\theta(p, q))q(p - \bar{v}) = m(\theta(p, 1))(p - \bar{v})$, so $\theta(p, q) \geq \theta(p, 1)$, and I have $0 > n(\theta(p, q))q(\bar{f} - p) - k$, so $(p, q) \in (\bar{p}, \infty) \times [0, 1]$ is not a profitable deviation.

Suppose that a coalition posts $p < \underline{p}$, with $q \in [0, 1]$. By Lemma 3, buyers expect type $\underline{v} = \underline{\delta f}$. Recall that $(\underline{f}, \underline{v})$ receives the complete information allocation, so $m(\theta(\underline{f}, \underline{v})(\underline{f} - \underline{v}) - k\theta(\underline{f}, \underline{v}) \geq m(\theta(p, q))q(\underline{f} - \underline{v}) - k\theta(p, q)$. So the zero-profit condition of buyers who trade with $(\underline{f}, \underline{v})$ in equilibrium gives

$$\begin{aligned}
0 &= m(\theta(\underline{f}, \underline{v}))(\underline{f} - p^*(\underline{v})) - k\theta(\underline{f}, \underline{v}) \\
&= m(\theta(\underline{f}, \underline{v}))(\underline{f} - \Pi(\underline{v})/m(\theta(\underline{f}, \underline{v})) - \underline{v}) - k\theta(\underline{f}, \underline{v}) \\
&= -\Pi(\underline{v}) + m(\theta(\underline{f}, \underline{v}))(\underline{f} - \underline{v}) - k\theta(\underline{f}, \underline{v}) \\
&\geq -\Pi(\underline{v}) + m(\theta(p, q))q(\underline{f} - \underline{v}) - k\theta(p, q) \\
&= -m(\theta(p, q))q(p - \underline{v}) + m(\theta(p, q))q(\underline{f} - \underline{v}) - k\theta(p, q) \\
&= m(\theta(p, q))q(\underline{f} - p) - k\theta(p, q).
\end{aligned}$$

Dividing both sides by $\theta(p, q)$ gives $0 \geq n(\theta(p, q))q(\underline{f} - p) - k$, so $(p, q) \in [0, \underline{p}] \times [0, 1]$ is not a profitable deviation, and the proof is complete. □

Proof of Corollary 1 These follow directly from differentiating the expressions for p^*, $m(\theta^*)q^*$, and $\tilde{\theta}(f, v)$ in Proposition 2 and the strict convexity of the profit function $\Pi(v)$. □

Proof of Corollary 2 These follow directly from differentiating the expressions for $\tilde{\theta}$ and $\tilde{q}$ in Proposition 2. □

Proof of Proposition 2 (i) and (ii): As p increases, v increases (Cor. 1), so $m(\theta)q$ decreases. If q is fixed, then θ must decrease, and vice versa. (iii): Fixing p is equivalent to fixing v (Cor. 1), so apply Cor. 2. □

Proof of Proposition 3.

Part (i): Given p, Corollary 1 implies that v is fixed, so apply Corollary 2.

Part (ii): Denote the liquidity and quantity sold over domain $\tilde{S}$ by $\tilde{\theta}$ and $\tilde{q}$, and over domain S by θ and q. Denote $\bar{S} \equiv \{(f, \delta) \in S : \delta f > \bar{\delta}\underline{f}\}$ and $\underline{S} \equiv \{(f, \delta) \in S : \delta f < \bar{\delta}\underline{f}\}$. Then $\theta(f, \delta) = \tilde{\theta}(f, \delta f)$ and $q(f, \delta) = \tilde{q}(f, \delta f)$. This gives

$$\begin{aligned}
\theta_1 &= \tilde{\theta}_1 + \tilde{\theta}_2\delta \\
&= -\frac{1}{k}[\Pi'(v) + \delta\Pi''(v)(f - v)].
\end{aligned}$$

Also note that differentiating (2) with respect to v gives

$$\left[m'(\underline{\theta}(v))(\underline{f}(v) - v) - k\right]\Pi''(v) = -\Pi'(v)m'(\underline{\theta}(v))\underline{f}'(v), \qquad \text{(A.3)}$$

where I denote $\underline{\theta}(v) \equiv \tilde{\theta}(\underline{f}(v), v)$ as the liquidity of the lowest quality asset f for a given private valuation v, and therefore the argument of m' in (A.3) and of m in (2) is $\underline{\theta}(v)$.

First, I show that if $\delta f > \bar{\delta}\underline{f}$, then liquidity is strictly decreasing in asset quality f. Use (A.3) to solve for $\Pi''(v)$, note that if $\delta f > \bar{\delta}\underline{f}$, then $\underline{f}(v) = v/\bar{\delta}$. Rearrange the above equation to get

$$\theta_1 = \frac{-\Pi'(v)}{k[m'(\underline{\theta}(v))(1-\bar{\delta})v/\bar{\delta} - k]}[-(\bar{\delta} - \delta)m'(\underline{\theta}(v))v/\bar{\delta} - k] < 0.$$

Next, I show that for high enough matching efficiency r, if $\delta f < \bar{\delta}\underline{f}$, then liquidity is strictly increasing in asset quality. Consider $\theta_1(f, \delta)$, which is the liquidity partial derivative with respect to f. Recall that $k\theta(f, \delta) = -\Pi(\delta f) - \Pi'(\delta f)(f - \delta f)$. Differentiating with respect to f gives

$$k\theta_1(f, \delta) = -\Pi'(\delta f) - \Pi''(\delta f)(1-\delta)\delta f \qquad \text{(A.4)}$$

For a given v, denote the liquidity associated with the lowest type $\underline{f}(v)$ as $\underline{\theta}(v) \equiv \tilde{\theta}(\underline{f}(v), v)$. In the low value region $\underline{S} = \{(f, \delta) : \delta f < \bar{\delta}\underline{f}\}$, $\underline{f}(v) = \underline{f}$, so $\underline{\theta}(v) = \tilde{\theta}(\underline{f}, v)$ and $\tilde{q}(\underline{f}, v) = 1$. Then I can compute $\Pi''(v)$ by noting that $\Pi'(v) = -m(\underline{\theta}(v))$. It can be easily shown that because $\underline{f}(v) = \underline{f}$ in $\underline{S}$, then types on the left border receive the complete information allocation, so $m'(\underline{\theta}(v))(\underline{f} - v) = k$. This gives $\Pi''(v) = -m'(\underline{\theta}(v))\underline{\theta}'(v) = -\frac{[m'(\underline{\theta}(v))]^2}{m''(\underline{\theta}(v))(\underline{f}-v)}$. Returning to (A.4), I have

$$k\theta_1(f, \delta) = m(\underline{\theta}(v)) + \delta\frac{[m'(\underline{\theta}(v))]^2}{m''(\underline{\theta}(v))}\frac{(f-v)}{(\underline{f}-v)}.$$

Dividing both sides by $m(\underline{\theta}(v))$ and incorporating the assumed form of $m(\theta) = (1+\theta^{-r})^{-1/r}$, I have that $\theta_1(\underline{f}, \delta) > 0$ if and only if $(1+r)\underline{\theta}(v)^r > \delta(f-v)/(\underline{f}-v)$. Use $m'(\underline{\theta}(v))(\underline{f} - v) = k$ to solve for $\underline{\theta}(v)$, plug in the previous inequality, and rearrange to get that $\theta_1(f, \delta) > 0$ if and only if

$$\left(\frac{\underline{f}(1-\delta)}{k}\right)^{\frac{r}{1+r}} > \frac{\delta}{1+r}\frac{f-v}{\underline{f}-v} + 1. \qquad \text{(A.5)}$$

By assumption, $\underline{f}(1-\delta)/k > 1$, so as $r \to \infty$, the limit of the left hand side of (A.5) is strictly greater than the limit of the righthand side, and therefore $\theta_1(f, \delta) > 0$ for high enough r. □

B Liquidity Converges to Retention

The equilibrium of Proposition 1 is similar to the retention equilibrium of DeMarzo and Duffie 1999, in which there are no search frictions and higher quality sellers separate by offering a lower quantity q. The following theorem shows just how similar these two equilibria are: as participation costs k in my model converge to zero, the equilibrium converges to an equilibrium equivalent to that in DeMarzo and Duffie 1999.

Theorem B.1. *As participation costs $k \to 0$,*

(i) $p^*(f) \to f$

(ii) $m(\Theta^*(f)) \to \left(\frac{f}{\underline{f}}\right)^{\frac{1}{1-\delta}}$

(iii) $P^*(m(\theta)) \to \frac{\underline{f}}{[m(\theta)]^{1-\delta}}$,

which is equivalent to the separating equilibrium in DeMarzo and Duffie 1999 if $m(\theta)$ is replaced by quantity sold q.

Proof. For ease of notation, I drop the * on all equilibrium functions, use $\theta(f)$ in place of $\Theta^*(f)$, and let $\underline{\theta} \equiv \Theta^*(\underline{f})$.

Part (i): The price function may be written as

$$P(\theta(f)) = f - \frac{k\theta(f)}{m(\theta(f))},$$

so I must show that the discount $k\theta/m(\theta)$ goes to zero as k goes to zero.

First consider the equilibrium for the lowest type $\underline{f}$. Recall that $\underline{\theta}$ solves the complete information FOC $m'(\underline{\theta})(1-\delta)\underline{f} = k$, so $\underline{\theta} \to \infty$ as $k \to 0$. Using the FOC, I can express the discount for $\underline{f}$ as

$$\frac{k\underline{\theta}}{m(\underline{\theta})} = (1-\delta)\underline{f}\frac{\underline{\theta}m'(\underline{\theta})}{m(\underline{\theta})}.$$

The following lemma guarantees that this discount goes to zero.

Lemma B.1.

$$\lim_{\theta \to \infty} \theta m'(\theta) = 0$$

Proof. Observe that due to the concavity of m, for any $\hat{\theta} > 0$, if $\theta > \hat{\theta}$, then

$$m'(\theta) < \frac{m(\theta) - m(\hat{\theta})}{\theta - \hat{\theta}}.$$

If so, then for any $\hat{\theta} > 0$,

$$\lim_{\theta \to \infty} \theta m'(\theta) \leq \lim_{\theta \to \infty} \theta \left[\frac{m(\theta) - m(\hat{\theta})}{\theta - \hat{\theta}} \right] = \lim_{\theta \to \infty} \frac{m(\theta) - m(\hat{\theta})}{1 - \frac{\hat{\theta}}{\theta}} = 1 - m(\hat{\theta})$$

This holds for any $\hat{\theta}$, and since $\sup_{\hat{\theta}} m(\hat{\theta}) = 1$, I must have $\lim_{\theta \to \infty} \theta m'(\theta) = 0$. □

Now recall from monotonicity that $\theta(f) < \underline{\theta}$ for any $f > \underline{f}$, so as $k \to 0$,

$$0 < \frac{k\theta}{m(\theta)} < \frac{k\underline{\theta}}{m(\underline{\theta})} = (1 - \delta)\underline{f} \frac{\underline{\theta} m'(\underline{\theta})}{m(\underline{\theta})} \to 0.$$

By the squeeze theorem, the discount $\frac{k\theta}{m(\theta)}$ for any f goes to zero, and therefore $P(\theta(f)) \to f$.

Part (ii): Let $\Theta(f, k)$ be the equilibrium θ for a given f and k. Let $h(\theta, k)$ be the inverse of $\Theta(f, k)$ so that $h(\Theta(f, k), k) = f$. If so, then $\Theta_1(h(\theta, k), k) = \frac{1}{h_1(\theta, k)}$. Then write (1) in terms of h:

$$\Big(m'(\theta)(1 - \delta)h(\theta, k) - k \Big) = -m(\Theta) h_1(\theta, k),$$

Using integrating factors, I can solve for h explicitly:

$$h(\theta, k) = m(\theta)^{-(1-\delta)} \left(k \int m(\theta)^{-\delta} d\theta + C(k) \right),$$

where

$$C(k) = \underline{f} m(\underline{\theta}(k))^{(1-\delta)} - k \int m(\theta)^{-\delta} d\theta \bigg|_{\theta=\underline{\theta}(k)}$$
$$= \underline{f} m(\underline{\theta})^{(1-\delta)} - (1-\delta)\underline{f} m'(\underline{\theta}) \int m(\theta)^{-\delta} d\theta \bigg|_{\theta=\underline{\theta}}$$

Note that because $\lim_{\theta\to\infty} m(\theta)^{-\delta} = 1$, it must be that $\lim_{\underline{\theta}\to\infty} \int m(\theta)^{-\delta} d\theta\big|_{\theta=\underline{\theta}} = \infty$. Apply L'Hopital's Rule first to Lemma B.1 and then to the second term of $C(k)$ to show that the second term converges to zero, which implies $C(k) \to \underline{f}$. This implies that as $k \to 0$, $h(\theta, k) \to m(\theta)^{-(1-\delta)}\underline{f} \equiv h(\theta)$. So then

$$P(\theta, k) = h(\theta, k) - \frac{k\theta}{m(\theta)} \to h(\theta) = \frac{\underline{f}}{[m(\theta)]^{1-\delta}}.$$

Part (iii): Note that $h(\theta)$ is invertible, so let $\Theta(f)$ be its inverse. Because $h(\Theta(f,k),k) = f$, it must be that $\Theta(f,k) \to \Theta(f)$. By the continuity of m,

$$\lim_{k\to 0} m(\Theta(f,k)) = m(\Theta(f)) = \left(\frac{\underline{f}}{h(\Theta(f))}\right)^{\frac{1}{1-\delta}} = \left(\frac{\underline{f}}{f}\right)^{\frac{1}{1-\delta}}.$$

□

The first item of the theorem is intuitive; as participation costs go to zero, buyers require no discount in order to compensate them for searching, so prices converge to the inherent value of the asset. The second item illustrates the equivalence of liquidity (probability of sale) in my model to retention (quantity sold) in the model of DeMarzo and Duffie 1999. A seller with any particular asset quality f will sell the asset, in the limit, with the same probability $m(\Theta^*(f))$ as his quantity sold $Q^*(f)$ in the model of DeMarzo and Duffie 1999. The third item follows from the first two, and is included simply for completeness in order show that the relation between sale probability $m(\theta)$ and price $P^*(m(\theta))$ is the same as DeMarzo and Duffie 1999's formula relating price to quantity sold q.

The theorem is reassuring because it corresponds with our intuition that screening sellers with liquidity (probability of sale) is analogous to screening with quantity (quantity unsold); indeed, in the limit, the two instruments are equivalent.

C Frictionless Search

This section demonstrates the significance of search frictions for the results of the paper. The absence of search frictions is formalized with the piecewise linear matching function $m(\theta) = \min[\theta, 1]$, which implies that the short side of the market matches with certainty. The first result identifies the special conditions that must hold in order for, under one-dimensional private information, retention and liquidity to be interchangeable sorting instruments.

Proposition C.1. *If buyers searching for q units of an asset must pay participation costs kq, and matching is frictionless, then any allocation $(p(f), \theta(f), q(f))$ satisfying $p(f) = f - k$, $\theta(f) \le 1$, and $\theta(f)q(f) = \phi(f)$, where $\phi(\cdot)$ solves*

$$\phi(f) + \Big((1-\delta)f - k\Big)\phi'(f) = 0 \qquad \phi(\underline{f}) = 1,$$

is an equilibrium.

Proof. To show that seller optimality is satisfied, I show that the allocation $(p(f), \theta(f), q(f))$ is a mechanism that satisfies GIC. The solution $\phi(\cdot)$ to the ODE must be differentiable and therefore continuous, so $\Pi(f) = m(\theta(f))q(f)(p(f) - \delta f) = \phi(f)(f - k - \delta f)$ is continuous. Differentiating the above expression for $\Pi(\cdot)$ and applying the ODE in the proposition gives $\Pi'(f) = \phi'(f)[(1-\delta)f - k] + \phi(f)(1-\delta) = -\delta\phi(f) = -\delta\theta(f)q(f) = -\delta m(\theta(f))q(f)$, and inspection of the ODE reveals that $\phi'(f) < 0$, so $m(\theta(f))q(f)$ is strictly decreasing. So by Lemma 1, GIC is satisfied.

To show that buyers earn zero profit in equilibrium, note that $\theta(f) \le 1$ implies $n(\theta) = 1$, so $n(\theta(f))(f - p(f)) = f - p(f) = k$. To show that off-equilibrium (p, q) are not profitable, suppose that buyers post (p', q'), where there exists some f such that $p' = p(f)$, but $q' \neq q(f)$. Then by Lemma 3, buyers expect type f, so $n(\theta(p', q'))(f - p') \leq f - p' = f - p(f) = k$. If there does not exist some f such that $p' = p(f)$, then suppose $p' > p(\bar{f})$. Then by Lemma 3, buyers expect type $\bar{f}$, so then $n(\theta(p', q'))(\bar{f} - p') \leq \bar{f} - p' < \bar{f} - p(\bar{f}) = k$. If $p' < p(\underline{f})$, then by Lemma 3, if there exists a type f such that $\theta(p', q', f) < \infty$, it must be type $\underline{f}$. But $q'(p' - \delta\underline{f}) < \phi(\underline{f})(p(\underline{f}) - \delta\underline{f}) = \Pi(\underline{f})$, contradicting $\theta(p', q', \underline{f}) < \infty$. So $\theta(p', q') = \infty$, and therefore (p', q') is not a profitable deviation. □

In Proposition 1, the strict concavity of the matching function (i.e., fundamental search frictions) is what distinguishes liquidity screening from retention screening. In contrast, Proposition C.1 shows that without fundamental search frictions, if participation costs scake with quantity, then the two instruments are equivalent.

I now consider the impact of frictionless search on the existence of a two-dimensional fully separating equilibrium. I show that a frictionless matching function of the form $m(\theta) = \min[\theta, 1]$ conflicts with the assumed rectangular type space $[\underline{f}, \bar{f}] \times [\underline{\delta}, \bar{\delta}]$, preventing full separation. The key point is that frictionless search implies buyers match with certainty, shutting down a degree of freedom for satisfying buyer zero-profit along the left border of the type space. However, for an arbirarily small modification of the type space, frictionless search is compatible with full separation, and the main equilibrium characterized in the paper is preserved.

Proposition C.2. *If the matching function is frictionless, so that* $m(\theta) = \min[\theta, 1]$*, then*

(i) over the rectangular type space $[\underline{f}, \bar{f}] \times [\underline{\delta}, \bar{\delta}]$*, there does not exist a fully separating equilibrium, but*

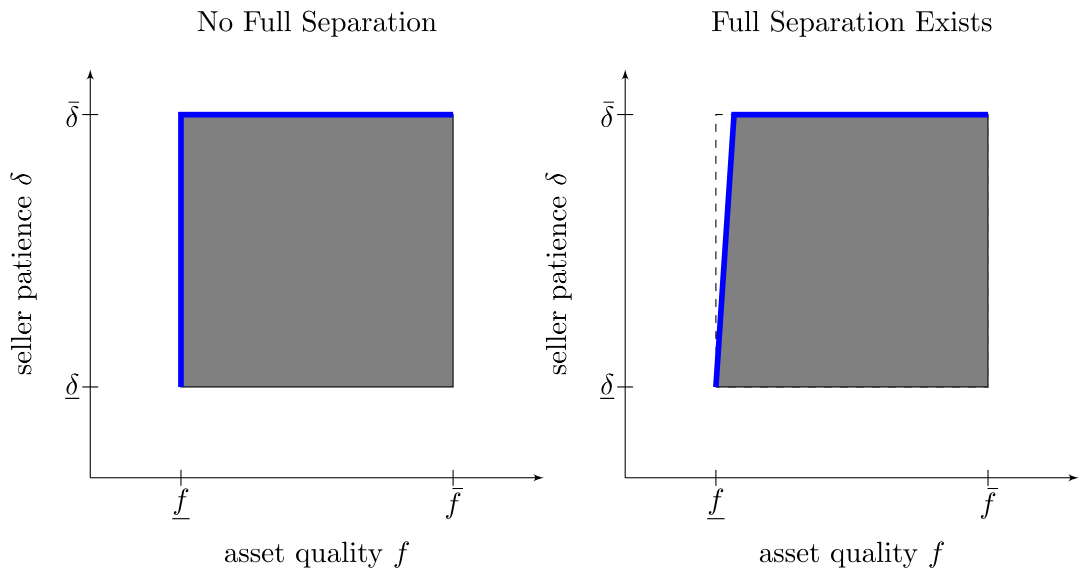
(ii) if the type space is modified so that $\underline{f}(v)$ *is strictly increasing in* v*, then there exists one and only one fully separating equilibrium. This equilibrium is characterized by the equations in Theorem 2.*

Proof. Part (i): By Lemma 1, for almost every $v \in V$, $\Pi'(v)$ exists and satisfies $-\Pi'(v) = m(\theta(f,v))q(f,v)$. So for almost all $v \in V$, $\Pi(v) = m(\theta(f,v))q(f,v)(p(f,v) - v) = -\Pi'(v)(p(f,v) - v)$, and therefore $p(\cdot, v)$ is constant. If the equilibrium is fully separating, then $E[\tilde{f}|p(f,v), q(f,v)] = f$, so buyer zero profit gives $k = n(\theta(f,v))q(f,v)[f - p(v)] = m(\theta(f,v))q(f,v)[f - p(v)]/\theta(f,v)$, which implies $\theta(f,v)k = -\Pi'(v)[f - p(v)]$. Therefore, $\theta(f,v)$ is strictly increasing in f, and full separation implies $q(f,v) = -\Pi'(v)/m(\theta(f,v))$ is strictly decreasing in f, which implies $\theta(f,v) \leq 1$. Then for almost every $v \in V$, $n(\theta(f,v)) = 1$, so buyer zero profit implies $k = q(\underline{f},v)(\underline{f} - p(\underline{f},v))$. By Lemma 7, $p(\underline{f},\cdot)$ is strictly increasing, and therefore there exist v, v' with $v < v' \leq \bar{\delta}\underline{f}$ such that $q(\underline{f},v) = k/(\underline{f} - p(\underline{f},v)) < k/(\underline{f} - p(\underline{f},v') = q(\underline{f},v') \leq 1$. By Lemma 8, this is a contradiction.

Part (ii): If $\underline{f}(v)$ is strictly increasing, then Lemmas A.1, 2, 3 (with the modification that there exists a type v such that $\theta(p,q,v) < 1$), 7, and 8 hold, and so does the proof of the differential equation.

To show the boundary condition, suppose to the contrary that $p(\underline{f},\underline{v}) \neq p_{CI}(\underline{f},\underline{v})$. The argument that $p_{CI}(\underline{f},\underline{v})$ is a deviation is the same as in the proof of Proposition 2, except that $\underline{\theta}'(v) < 0$ follows from $\underline{f}'(v) > 0$ rather than from buyer zero profit. This argument also shows that $p_{CI}(\underline{f},\underline{v}) < p(f,v)$ for all $(f,v) \in \tilde{S}$ with $v > \underline{v}$. In addition, observe that $p(\underline{f},\underline{v}) = \underline{f} - k/n(\underline{\theta}(\underline{v})) \leq \underline{f} - k = \underline{f} - k/n(\theta_{CI}(\underline{v})) = p_{CI}(\underline{f},\underline{v})$. So if $p(\underline{f},\underline{v}) \neq p_{CI}(\underline{f},\underline{v})$, it must be that $p(\underline{f},\underline{v}) < p_{CI}(\underline{f},\underline{v})$, and therefore $\underline{f} - k/n(\underline{\theta}(\underline{v})) < \underline{f} - k/n(\theta_{CI}(\underline{v}))$, which implies $\underline{\theta}(\underline{v}) > \theta_{CI}(\underline{v}) = 1$.

Consider the deviation $(p', 1)$, where $p' \in (p(\underline{f},\underline{v}), \inf_{\tilde{S}} p(f,v))$. By Lemma 3, buyers expect type $(\underline{f},\underline{v})$, so $\theta(p',1)$ satisfies $m(\theta(p',1))(p' - \underline{v}) = \Pi(\underline{v}) = m(\underline{\theta}(\underline{v}))(p(\underline{f},\underline{v}) - \underline{v})$, which implies $m(\theta(p',1)) < m(\underline{\theta}(\underline{v})) = 1$. But then $\theta(p',1) < 1 < \underline{\theta}(\underline{v})$, so $n(\theta(p',1)) > n(\underline{\theta}(\underline{v})$, which implies $n(\theta(p',1))(\underline{f} - p(\underline{f},\underline{v})) > n(\underline{\theta}(\underline{v}))(\underline{f} - p(\underline{f},\underline{v})) = k$. So there exists p' close enough to $p(\underline{f},\underline{v})$, such that $n(\theta(p',1))(\underline{f} - p') > k$, which proves that $(p',1)$ is a profitable deviation, the desired contradiction. So $p(\underline{f},\underline{v})) = p_{CI}(\underline{f},\underline{v})$, which implies that $\underline{f} - k/n(\underline{\theta}(\underline{v})) = \underline{f} - k/n(\theta_{CI}(\underline{v}))$, and therefore $\underline{\theta}(\underline{v}) \leq 1$.



(a) Original rectangular type space.

(b) Modified type space

Figure 1: The figure illustrates Proposition C.2, showing that under the rectangular type space, frictionless search implies no fully separating equilibrium exists. But for an arbitrarily small modification of the type space, frictionless search is compatible with full separation. The key modification is that the worst asset quality $\underline{f}(v)$ must be strictly increasing in seller private value.

Finally, to show that $\underline{\theta}(\underline{v}) = \theta_{CI}(\underline{f}, \underline{v}) = 1$, suppose not. Then from the above, $\underline{\theta}(\underline{v}) < 1$. So if buyers deviate by posting $p' < p_{CI}(\underline{f}, \underline{v})$, close enough to $p_{CI}(\underline{f}, \underline{v})$, then $m(\theta(p', 1))(p' - \underline{v}) = \Pi(\underline{v}) = m(\underline{\theta}(\underline{v}))(p_{CI}(\underline{f}, \underline{v}) - \underline{v})$, with $\theta(p', 1) \in (\underline{\theta}(\underline{v}), 1)$. But then $n(\theta(p', 1))(\underline{f} - p') > n(\underline{\theta}(\underline{v}))(\underline{f} - p(\underline{f}, \underline{v})) = k$, so $(p', 1)$ is a profitable deviation, a contradiction. Therefore $\underline{\theta}(\underline{v}) = 1 = \theta_{CI}(\underline{f}, \underline{v})$. □

The key difference made by frictionless search is that buyers match with certainty.[1] The intuition for part (i) is easiest to see by considering sellers with

1. Incentive compatibility and full separation require variation in the sellers' matching probability $m(\theta)$, which can only vary if $\theta < 1$. This implies buyers match with probability $n(\theta) = 1$.

the worst assets $f = \underline{f}$, which occupy the left boundary in Figure 1a.

The problem is that as we move up along the left boundary, seller values v rise, so full separation requires the price p to strictly increase. By buyer zero-profit, this increasing price must be offset by either increasing asset quality f or increasing trade volume $n(\theta)q$. If the set of types (f, δ) is rectangular as in Figure 1a, then asset quality along the left boundary is constant at $\underline{f}$; this leaves only volume $n(\theta)q$ to compensate buyers for the increasing price, and if search is frictionless, then volume is simply quantity q. But if quantity q increases along the left boundary, then buyers have the incentive to deviate by offering impatient sellers a higher quantity $q' > q$, so this can't be an equilibrium.

However, Part (ii) of the proposition shows that full separation is not really so fragile: it is the strong assumption of a rectangular type space that, along with frictionless search, prevents separation. Part (ii) shows that full separation is possible under an arbitrarily small modification of the type space, so that the worst asset quality $\underline{f}(v)$ is strictly increasing in seller value v. For example, if the set of types (f, δ) is a trapezoid with a slightly positively sloped left boundary, as depicted in Figure 1b, then buyers trading with sellers along the left boundary are compensated for the increasing price with a higher asset quality f; in this way, buyer zero-profit is preserved even if they match with certainty and purchase the full quantity $q = 1$ from the sellers.

D Example: Holding Cost

The methodology in the paper may be applied to a general class of problems in which the seller has private information about both his own value and the buyer's value. For example, the seller's private value may be a combination of the asset quality and a holding cost, as in Chang 2018. This section studies the implications if the sellers have a holding cost c, rather than patience δ.

Consider a seller who holds an asset at date 0 that pays $f \in [\underline{f}, \bar{f}]$ at date

1. The seller has a holding cost $c \in [\underline{c}, \bar{c}]$ that scales with the quantity of the asset owned. Buyers post prices p and quantities q in a competitive search market, and sellers choose a pair (p, q) at which to search for buyers. A sellers with asset quality f and holding cost c who searches in a market with terms (p, q) earns expected profit

$$(1 - m(\theta(p,q))[f - c] + m(\theta(p,q))[qp + (1-q)(f-c)]$$
$$= [f - c] + m(\theta(p,q)q(p - (f-c)).$$

The chosen trade terms (p, q) affect only the second term, so sellers choose (p, q) to maximize

$$m(\theta(p,q))q(p - (f-c)).$$

It is clear from the form of the seller's payoff function that most results are the same as if seller patience was formalized with a discount factor δ, as in the text of the paper. That is, if we define $v = f - c$, most proofs go through the same as if v represents δf. There are two minor exceptions, though, which I explain below.

The first exception is the complete information market tightness θ. Assuming a holding cost c, complete information market tightness solves $m'(\theta)c = k$, and is therefore invariant to asset quality f, as in Chang 2018. But this does not qualitatively affect the shape of asymmetric information equilibria, because under asymmetric information, only types holding the worst assets $\underline{f}$ receive the complete information allocation.

The second exception is Proposition 3 part (ii), which shows that for a fixed patience δ, liquidity θ may be nonmonotonic. Assuming a holding cost c instead of patience δ, the result holds, though some expressions in the proof are slightly different. I include the modified proof below for completeness.

Proof of Proposition 3 part (ii), assuming holding cost. Denote the liquidity and quantity sold over domain $\tilde{S}$ by $\tilde{\theta}$ and $\tilde{q}$, and over domain S

by θ and q. Then $\theta(f,c) = \tilde{\theta}(f, f-c)$ and $q(f, f-c) = \tilde{q}(f, f-c)$. This gives

$$\begin{aligned}\theta_1 &= \tilde{\theta}_1 + \tilde{\theta}_2 \\ &= -\frac{1}{k}[\Pi'(v) + \Pi''(v)(f-v)].\end{aligned}$$

Also note that differentiating (2) with respect to v gives

$$\left[m'(\underline{\theta}(v))(\underline{f}(v) - v) - k\right] \Pi''(v) = -\Pi'(v)m'(\underline{\theta}(v))\underline{f}'(v), \tag{D.1}$$

where I denote $\underline{\theta}(v) \equiv \tilde{\theta}(\underline{f}(v), v)$ as the liquidity of the lowest quality asset f for a given private valuation v, and therefore the argument of m' in (D.1) and of m in (2) is $\underline{\theta}(v)$.

First, I show that if $f - c > \underline{f} - \underline{c}$, then liquidity is strictly decreasing in asset quality. Use (D.1) to solve for $\Pi''(v)$, and note that if $f - c > \underline{f} - \underline{c}$, then $\underline{f}(v) = v + \underline{c}$. Rearrange the above equation to get

$$\theta_1 = \frac{-\Pi'(v)}{k[m'(\underline{\theta}(v))(\underline{f}(v) - v) - k]}[(\underline{c} - c)m'(\underline{\theta}(v)) - k] < 0.$$

Next, I show that if $f - c < \underline{f} - \underline{c}$, then liquidity is strictly increasing in asset quality. Recall that $k\theta(f,c) = -\Pi(f-c) - \Pi'(f-c)c$. Differentiating with respect to f gives

$$k\theta_1(f,c) = -\Pi'(f-c) - \Pi''(f-c)c \tag{D.2}$$

For a given v, denote the liquidity associated with the lowest type $\underline{f}(v)$ as $\underline{\theta}(v) \equiv \tilde{\theta}(\underline{f}(v), v)$. In the low value region $\underline{S} = \{(f,c) : f - c < \underline{f} - \underline{c}\}$, $\underline{f}(v) = \underline{f}$, so $\underline{\theta}(v) = \tilde{\theta}(\underline{f}, v)$. Then I can compute $\Pi''(v)$ by noting that $\Pi'(v) = -m(\underline{\theta}(v))$. It can be easily shown that because $\underline{f}(v) = \underline{f}$ in $\underline{S}$, types on the left border receive the complete information allocation, so $m'(\underline{\theta}(v))(\underline{f} - v) = k$. This gives $\Pi''(v) = -m'(\underline{\theta}(v))\underline{\theta}'(v) = -\frac{[m'(\underline{\theta}(v))]^2}{m''(\underline{\theta}(v))(\underline{f}-v)}$. Returning to (D.2), I have

$$k\theta_1(f,c) = m(\theta(\underline{v})) + \frac{[m'(\underline{\theta}(v))]^2}{m''(\underline{\theta}(v))} \frac{f-v}{\underline{f}-v}.$$

Dividing both sides by $m(\underline{\theta}(v))$ and incorporating the assumed form of $m(\theta) = (1+\theta^{-r})^{-1/r}$, I have that $\theta_1(f,c) > 0$ if and only if $(1+r)\underline{\theta}(v)^r > (f-v)/(\underline{f}-v)$, where $v = f - c$. Use $m'(\underline{\theta}(v))(\underline{f} - v) = k$ to solve for $\underline{\theta}(v)$, plug in the previous inequality, and rearrange to get that $\theta_1(f,c) > 0$ if and only if

$$\left(\frac{\underline{f} - v}{k}\right)^{\frac{r}{1+r}} > \frac{1}{1+r}\frac{f - v}{\underline{f} - v} + 1, \tag{D.3}$$

where $v = f - c$. By assumption, $(\underline{f} - v)/k > 1$ in $\underline{S}$, so as $r \to \infty$, the limit of the left hand side of (D.3) is strictly greater than the limit of the righthand side, and therefore $\theta_1(f,c) > 0$ for high enough r. □

E Additional Fire Sale Results

Proposition E.1. *The results in Proposition 4 part (ii) hold regardless of whether the average is computed per offer, per sale, quantity-weighted per offer, or quantity-weighted per sale.*

Proof. Proposition 4 computes the average per offer. For a fixed price p, the seller value v is also fixed, so the total number of sales that occur at price p is $g(v)\int_{\underline{\delta}(v)}^{\bar{\delta}(v)} m(\tilde{\theta}(v\delta^{-1}, v))g(\delta|v)d\delta$. As a result, the average per sale may be computed by integrating over the measure

$$g_s(\delta|v) \equiv \frac{m(\tilde{\theta}(v\delta^{-1}, v))g(\delta|v)}{\int_{\underline{\delta}(v)}^{\bar{\delta}(v)} m(\tilde{\theta}(v\delta^{-1}, v))g(\delta|v)d\delta},$$

where I have cancelled $g(v)$ from the numerator and denominator. Similarly, the quantity-weighted average per sale may be computed by integrating over the measure

$$g_{qs}(\delta|v) \equiv \frac{\tilde{q}(v\delta^{-1}, v)m(\tilde{\theta}(v\delta^{-1}, v))g(\delta|v)}{\int_{\underline{\delta}(v)}^{\bar{\delta}(v)} \tilde{q}(v\delta^{-1}, v)m(\tilde{\theta}(v\delta^{-1}, v))g(\delta|v)d\delta}$$

And the quantity-weighted average per offer may be computed by integrating over the measure

$$g_q(\delta|v) \equiv \frac{\tilde{q}(v\delta^{-1}, v)g(\delta|v)}{\int_{\underline{\delta}(v)}^{\bar{\delta}(v)} \tilde{q}(v\delta^{-1}, v)g(\delta|v)d\delta}.$$

Define $\hat{g}_s$, $\hat{g}_{qs}$, and $\hat{g}_q$ analogously by replacing $g(\delta|v)$ with $\hat{g}(\delta|v)$ in each expression above. It can be easily shown that MLRP of $g(\delta|v)$ and $\hat{g}(\delta|v)$ implies MLRP and therefore FOSD of the weighted measures defined above. From here, the proof of Proposition 4 part (ii) may be immediately applied. □

F Comparison with Partial Pooling

Guerrieri and Shimer 2018 consider two dimensional private information in a similar framework as this model. One crucial difference is that they do not include the retention signal q in their framework, and therefore are unable to fully separate both asset quality and seller impatience. This leads to a partial pooling equilibrium in which sellers are distinguishable only up to their private valuation δf.

This section compares the fully separating equilibrium of my model to a partial pooling equilibrium analogous to their model. My setting is not identical to the setup in Guerrieri and Shimer 2018, but the environments are sufficiently similar to offer instructive comparisons. I first solve for partial pooling equilibrium, and then show that it Pareto dominates full separation.

Proposition F.1. *Suppose that $E[\tilde{f}|v]$ is strictly increasing in v, and that when buyers post off-equilibrium terms $(p,q) \notin M$, they expect all types willing to accept the lowest market tightness:* $\arg\inf_{(f',\delta')} \theta(p,q,f',\delta')$.

Then in the class of equilibria in which sellers of common value v pool, there exists a unique equilibrium. It takes the following form:

$$\tilde{p}^*(f,v) = \frac{\Pi(v)}{-\Pi'(v)} + v, \qquad \tilde{q}^*(f,v) = 1,$$

where $\theta(f,v)$ and $\Pi(v)$ are characterized by

$$\tilde{\theta}(f,v) = -\frac{1}{k}\left[\Pi(v) + \Pi'(v)(E[\tilde{f}|\tilde{\delta}\tilde{f}=v] - v)\right],$$

$$\Pi'(v) = -m\left(-\frac{1}{k}\left[\Pi(v) + \Pi'(v)(E[\tilde{f}|\tilde{\delta}\tilde{f}=v] - v)\right]\right), \qquad \Pi(\underline{v}) = \Pi_{CI}(\underline{f}(\underline{v}), \underline{v}). \tag{F.1}$$

Proof. The assumptions of the proposition along with Lemma 3 effectively reduce the private information to one dimension. As a result, the proof is identical to that of Theorem 1, in which δ is public and only f is private. In particular, one can show that the price function is strictly increasing in v, that $q(v) = 1$ for all $v \in V$, and that $\theta(v)$ is continuous and attains the complete information allocation at $v = \underline{v}$. Continuity of $\theta(v)$ implies the differential equation in the proposition. □

The assumption of strictly increasing expected asset quality $E[\tilde{f}|\tilde{\delta}\tilde{f} = v]$ is required in Guerrieri and Shimer 2018, and guarantees that sellers of lower value v are tempted to mimic sellers of higher value v.

The key feature of this equilibrium is that every type attempts to sell the entire quantity of the asset, so quantity sold q is equal to 1 for all types. This leaves price p as the only distinguishing signal; the proposition shows that price is an invertible function of seller value v, so buyers separate sellers only up to v, as in Guerrieri and Shimer 2018. The next proposition compares this partial pooling equilibrium to the fully separating equilibrium.

Proposition F.2. *Let Σ be the Pareto optimal equilibrium in which all types fully separate. Let Φ be the Pareto optimal equilibrium in which partial pooling occurs in the following way: sellers with distinct private values δf separate, and sellers with common private values δf pool.*

Then partial pooling Φ Pareto dominates full separation Σ: $\Pi_\Phi(\underline{v}) = \Pi_\Sigma(\underline{v})$ and $\forall v > \underline{v}$, $\Pi_\Phi(v) > \Pi_\Sigma(v)$.

Proof. Clearly, both equilibria have the same initial condition $\Pi(\underline{v}) = \Pi_{CI}(\underline{f}(\underline{v}), \underline{v})$, and the only difference between (2) and (F.1) is that $E[\tilde{f}|\tilde{f}\tilde{\delta} = v]$ has been substituted for $\underline{f}(v)$. As long as some sellers of value v have assets better than $\underline{f}(v)$, it must be that $E[\tilde{f}|\tilde{f}\tilde{\delta} = v] > \underline{f}(v)$.

Now suppose that the two equilibria have the same profit $\Pi(v)$ for some v, and consider how $\Pi'(v)$ differs under the two equilibria. Let g be a placeholder for either expression $E[\tilde{f}|\tilde{f}\tilde{\tilde{\delta}} = v]$ or $\underline{f}(v)$, and let α be a placeholder for $\Pi'(v)$ and write

$$\alpha = -m\left(-\frac{1}{k}\left[\Pi(v) + \alpha(g - v)\right]\right). \tag{F.2}$$

Now fix v and $\Pi(v)$, and consider how α changes as g increases from $\underline{f}(v)$ to $E[\tilde{f}|\tilde{f}\tilde{\tilde{\delta}} = v]$. Differentiate both sides of (F.2) with respect to g, and solve for $\alpha'(g)$ to obtain

$$\alpha'(g) = \frac{m'(...)(-\alpha)}{m'(...)(g - v) - k} > 0, \tag{F.3}$$

where the inequality is due to the fact that $\alpha = \Pi'(v)$ is negative and the denominator $m'(...)(g - v) - k$ is positive. Therefore, $\Pi'(v)$ is higher under partial pooling where $g = E[\tilde{f}|\tilde{f}\tilde{\tilde{\delta}} = v] > \underline{f}(v)$ than under full separation where $g = \underline{f}(v)$, so wherever $\Pi_\Phi(v)$ crosses $\Pi_\Sigma(v)$, it must be that $\Pi'_\Phi(v) > \Pi'_\Sigma(v)$.

Finally, suppose there exists a v at which the profit function $\Pi_\Phi(v)$ under pooling is less than or equal to that under full separation $\Pi_\Sigma(v)$. Then because the two equilibrium profit functions are equal at the initial condition $\Pi(\underline{v}) = \Pi_{CI}(\underline{f}, \underline{v})$, the pooling profit $\Pi_\Phi(v)$ must cross the separating profit $\Pi_\Sigma(v)$ from above, which contradicts $\Pi'_\Phi(v) > \Pi'_\Sigma(v)$. Therefore, for all $v > \underline{v}$, $\Pi_\Sigma(v) < \Pi_\Phi(v)$. $\square$

Figure 2 compares the expected profit $\Pi(v)$, expected quantity $m(\theta)q$, and price p under partial pooling to the same values under full separation. It also compares the average asset quality $E[f|v]$ and the worst asset quality $\underline{f}(v)$ among sellers of value v, assuming a uniform distribution over the type space $[\underline{f}, \bar{f}] \times [\underline{\delta}, \bar{\delta}]$. The figure shows that expected quantity $m(\theta)q$ may be higher under separation than under pooling among low value sellers. This is because, as explained in the text, in this region the incentive to mimic higher value sellers is very weak under full separation, because the worst asset $\underline{f}(v)$ (which constrains the payoff of all sellers of value v) is constant at $\underline{f}$ in that region. In contrast, under partial pooling, the average quality $E[f|v]$ is strictly increasing, so the incentive to mimic is greater, and expected quantity $m(\theta)q$ must decrease steeply in order to prevent mimicry.

Still though, the expected profit is always higher under partial pooling, because the pooling price in each class v is high enough to offset potentially low volume $m(\theta)q$. This occurs for two reasons. The first is that when the pooling volume sharply decreases, the price sharply increases in order to compensate higher value sellers for the lower volume, preventing them from mimicking sellers of lower value. The second reason is that under partial pooling, buyers pay a price commensurate to the average asset quality, which is strictly higher than the worst asset quality $\underline{f}(v)$ that determines the price under full separation.

It is not surprising that partial pooling improves the outcome for the worst quality $\underline{f}(v)$ in each class v, because sellers of the worst assets receive a higher price due to improved buyer expectations. But it may seem puzzling why the best types are not worse off under partial pooling compared to full separation, as is often the case in adverse selection models. The reason is that under full separation, the best types do not strictly benefit from being identified by the buyers. No single crossing condition exists along a fixed iso-value curve, so there is no way to give the best types a better outcome than the worst types without inducing mimicry by the worst types. As a result, under full separation, all types receive the same low profit as the worst type. Partial pooling on the other hand, by improving the profit for the worst types, improves the profit for all other types in the same class, and therefore Pareto dominates full separation.

Although partial pooling Pareto dominates full separation, it is less robust to off-equilibrium beliefs. Defining $T(p,q) = \arg\inf_{s\in\tilde{S}} \theta((p,q,),s)$, the standard restriction, which is used in the equilibrium definition, is given by:

$$\text{For any pair } (p,q) \notin M \text{ and type } s,\ \mu(s|p,q) = 0 \text{ if } s \notin T(p,q). \qquad \text{(R)}$$

There exist beliefs satisfying (R) which break partial pooling Φ and support full separation Σ, but not vice versa. The restriction (R) only pins down a unique private value v which buyers can expect to be attracted to a deviating

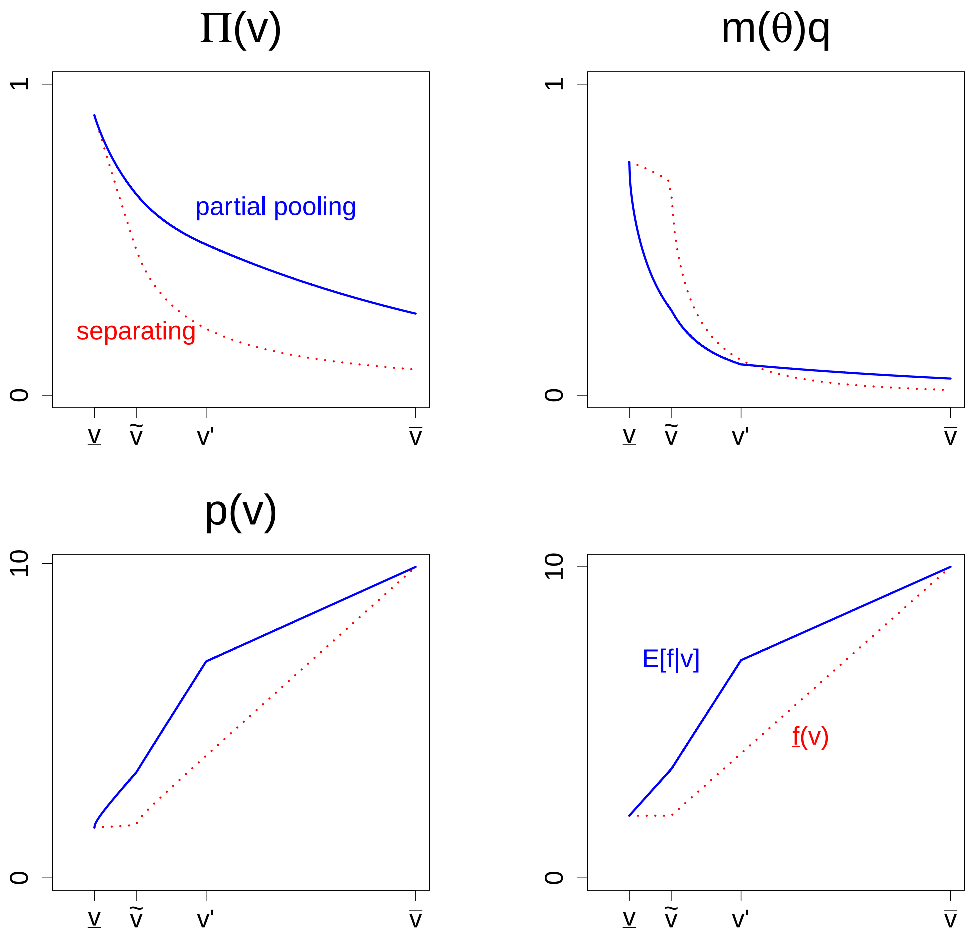


Figure 2: The figure shows the expected seller profit Π, expected quantity $m(\theta)q$, and price p as a function of the seller's private value $v = \delta f$, conditional on the equilibrium. In each subfigure, the solid blue line depicts the partial pooling equilibrium, and the dotted red line depicts the separating equilibrium. Here $\underline{v} = \underline{\delta}\underline{f}$, $\tilde{v} = \bar{\delta}\underline{f}$, $v' = \underline{\delta}\bar{f}$, $\bar{v} = \bar{\delta}\bar{f}$, and $E[\tilde{f}|\tilde{\delta}\tilde{f} = v]$ is computed assuming a uniform distribution over the type space $[\underline{f}, \bar{f}] \times [\underline{\delta}, \bar{\delta}]$.

contract (p, q). However, sellers of common private value v differ in asset quality f. If, for example, buyers expect the best asset quality $\bar{f}(v)$ among these sellers, then offering to buy a slightly smaller quantity $q < 1$ while receiving a significantly better asset $\bar{f}(v)$ is a profitable deviation; and therefore the pooling equilibrium is broken. This difficulty does not occur under full separation, however, because for a fixed equilibrium price p, the lowest equilibrium quantity q already corresponds to the highest asset quality $\bar{f}(v)$ among sellers who post p; so buyers stand to gain nothing by offering an even lower quantity q, which hurts both buyers and sellers.[2]

G Security Design

DeMarzo and Duffie 1999 solved their retention signaling equilibrium with a view toward studying security design; because my model builds on their framework, it is natural to examine whether the addition of search frictions and privately known patience alter the optimal security. As in DeMarzo and Duffie 1999, here I explore security design *before* the arrival of private information. See Li 2018 for a study of security design *after* the arrival of private information, in a setting of competitive search. In her setting, sellers may signal with the design of their securities, and in equilibrium they issue debt with many tranches of differing seniority.

Suppose that before sellers privately learn ther patience $\delta \in [\underline{\delta}, \bar{\delta}]$ as well as information related to the final payoff $X \geq 0$ of their asset, they can design a security $F = \phi(X) \in [0, X]$ to sell which is backed by that asset. Anticipating the arrival of private information and the fully separating equilibrium of the competitive search trading game discussed above, the seller designs the security

2. Guerrieri and Shimer 2018 justify their assumption that deviating buyers expect the average quality among sellers of common v by pointing out that those sellers have the same incentives. However, if we allow a kind of trembling hand irrationality in which only a subset of those sellers deviate so that their expected quality is higher than that of all sellers of common value v, then retention-free partial pooling may not be an equilibrium.

that will result in the highest expected payoff.

After the design of the security, but before the sale, the seller receives private information relevant to the payoff of the security. Denote the information by random variable $Z \in \mathbb{R}$, so that the issuer's conditional valuation of the security is $E(F|Z)$. For each security design F, the issuer assumes some liquidity schedule $\theta_F(p,q) : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$; if the seller posts price p, then $\theta_F(p)$ is the market liquidity of the security F. Given a security F, I can write the seller's objective as a function of terms (p,q), and therefore market tightness $\theta_F(p,q)$ as follows:

$$\delta E(X-F|Z) + [1-m(\theta_F(p,q))]\delta E(F|Z) + m(\theta_F(p,q))[(1-q)\delta E(F|Z) + qp]$$
$$= \delta E(X|Z) + m(\theta_F(p,q))q[p - \delta E(F|Z)].$$

After the seller not only designs the security, but also receives private information z and patience shock δ, then the seller's relevant private information consists of the particular outcome $f \equiv E[F(X)|z]$ of $E(F|Z)$ and the privately known patience. The seller's liquidation problem is

$$\Pi_F(f,\delta) = \max_{p>0, q\in[0,1]} m(\theta_F(p,q))q(p-\delta f). \tag{G.1}$$

Here, the equilibrium liquidation schedule $\theta_F(p,q)$ and therefore profit function $\Pi_F(f,\delta)$ depend on the structure of the security F. Before receiving private information Z, the seller anticipates this dependency, and designs the security F in order to induce the most favorable profit function $\Pi(f,\delta)$ to maximize his expected profit. Letting $V(F) \equiv E[\Pi_F(E(F|Z))]$ denote the sellers expected profit contingent on security F, the security design problem is

$$\sup_F V(F).$$

Before solving for the optimal F, I first define the following restriction on the conditional distribution of X given Z:

Definition G.1. *An outcome $\underline{z}$ of Z is a* uniform worst case *if, for any other outcome z and any interval $I \subset \mathbb{R}_+$ of outcomes of X,*

1. *if* $\mu(X \in I|z) > 0$, *then* $\mu(X \in I|\underline{z}) > 0$*;*

2. *the conditional of* $\mu(\cdot|z)$ *given* $X \in I$ *has first-order stochastic dominance over the conditional of* $\mu(\cdot|\underline{z})$ *given* $X \in I$*.*

Note that the existence of a uniform worst case is weaker than the monotone likelihood ratio property. I am now ready to solve for the optimal security F.

Proposition G.1. *If there is a uniform worst case of asset-relevant information, then even with search frictions and private information about seller patience* δ*, among increasing monotone securities, a standard debt contract* $F(X) = \min(X, d)$ *is an optimal security.*

Proof. Using a strategy identical to the proof of Proposition 10 in DeMarzo and Duffie (1999), I can show that given any increasing security G with $\underline{g} = E[G(X)|\underline{z}]$, then if $F(X) = \min[X, d]$ is a standard debt contract with $\underline{f} = \underline{g}$, then for all z, $g = E[G(X)|z] \geq E[F(X)|z] = f$. The profit function given in (2) depends on the region $[\underline{\delta}, \bar{\delta}]$ and the lower bound $\underline{f}$, and is strictly decreasing in δf. Of these parameters, ex ante the seller only has control over $\underline{f}$ when designing the security, so write the profit function as $\Pi(\delta f, \underline{f})$. Since $\Pi(\delta f, \underline{f})$ is decreasing in f for any $\delta \in [\underline{\delta}, \bar{\delta}]$, I have $\Pi(\delta g, \underline{g}) = \Pi(\delta g, \underline{f}) \leq \Pi(\delta f, \underline{f})$. Because this inequality holds for any δ and any z, take expectations to get $V(F) \geq V(G)$. So standard debt is an optimal security. □

The intuition here is similar to the intuition in DeMarzo and Duffie 1999. Intuitively, debt is the security least sensitive to private information on the underlying asset, so it minimizes the amount of information asymmetry, and therefore also the amount of *expected* retention required to credibly convince buyers that the asset is high value. Because standard debt is optimal for any public patience δ, debt must also be optimal for any privately observed random realization of δ, and therefore debt is optimal when patience δ is ex ante uncertain.

References

Chang, Briana. 2018. "Adverse selection and liquidity distortion." *The Review of Economic Studies* 85 (1): 275–306.

DeMarzo, Peter, and Darrell Duffie. 1999. "A liquidity-based model of security design." *Econometrica* 67 (1): 65–99.

Guerrieri, Veronica, and Robert Shimer. 2018. "Markets with multidimensional private information." *American Economic Journal: Microeconomics* 10 (2): 250–74.

Li, Qi. 2018. "Securitization and Liquidity Creation in Markets with Adverse Selection." Working Paper.